Business Calculus Test 1 Review Key

Dr. Graham-Squire, Spring 2017

1. Factor and simplify the expression $5x^2(3x^2+1)^4(6x)+(3x^2+1)^5(2x)$.

   Ans: $2x(3x^2+1)^4(18x^2+1)$

2. The average speed of a vehicle in miles per hour over a certain stretch of road, between 6 AM and 8 AM, is given by

   $$20t^2-40t+50 \qquad (0\le t\le 2)$$

   where t is in hours, with $t=0$ corresponding to 6 AM. Over what interval(s) of time is the average speed less than or equal to 35 mph?

   Ans: $\frac{1}{2}\le t\le \frac{3}{2}$

3. Dominic has decided to attach his baby sister Eva to a rocket. Suppose Dominic is standing 200ft from the rocket (over flat ground) and the rocket with Eva attached lifts off vertically in the air and reaches an altitude of x feet.

   (a) Find an expression (in terms of x) giving the distance between Dominic and Eva as the rocket goes up.

   (b) What is the distance between Dominic and Eva when the rocket reaches an altitude of 1,000 feet?

   Ans: (a) $d=\sqrt{x^2+40,000}$. (b) approximately 1,020 feet.

4. Find the equation of the line that passes through the points (1,2) and (-3,-2), and write the equation in the form $y=$ _____. Is the point (4, 8) also on that line? Explain why or why not.

   Ans: $y=x+1$. No, the point (4, 8) is not on the line because 4+1=5$\neq$ 8.

5. Sketch the graph of the function $f(x)=\begin{cases}4-x & \text{if } x<2\\ 2x-2 & \text{if } x\ge 2\end{cases}$

   Ans: Graph looks like a V with the point at $x=2$.

6. Find the rules for the composite functions $f(g(x))$ and $g(f(x))$ if $f(x)=\frac{x}{x^2+5}$ and $g(x)=\frac{1}{x^2}$. Simplify the resulting expressions.

   Ans: $f(g(x))=\frac{x^2}{5x^4+1}$ and $g(f(x))=\frac{x^4+10x^2+25}{x^2}$.

7. The sales of DVD players in year t (in millions of units) is given by the function

$$f(t) = 5.6(1+t) \qquad (0 \le t \le 3)$$

where $t = 0$ corresponds to 2001. Over the same period, the sales of VCRs (in millions of units) is given by

$$g(t) = \begin{cases} -9.6t + 22.5 & \text{if } 0 \le t \le 1 \\ -0.5t + 13.4 & \text{if } 1 < t \le 2 \\ -7.8t + 28 & \text{if } 2 < t \le 3 \end{cases}$$

(a) Show that more VCRs than DVD players were sold in 2001.

(b) When did sales of DVD players first exceed those of VCRs?

Ans: (a) $f(0) = 5.6$ and $g(0) = 22.5$, so there were more VCRs at $t = 0$, which is 2001.

(b) Setting $f(t)$ equal to the middle equation of $g(t)$ solves to $t = 1.28$. So DVDs first exceeded VCRs in early 2002.

8. Farmer Bob wants to enclose a rectangular area next to his barn with fencing. One side of the rectangular area will be the side of the barn, and the other three sides will be fencing. Suppose that Bob has 80 feet of fencing and his barn is 100 feet long.

(a) If the width of the rectangular area is x, find an expression for the area of the rectangle.

Ans: $A(x) = (80 - 2x)(x)$

(b) If the width is 10 feet, what will the area of the rectangle be?

Ans: 600 ft^2.

9. Find the limits. If the limit does not exist, write DNE and explain why.

(a) $\lim\limits_{x \to 2} \dfrac{x^2 + 4x - 12}{x^2 - 2x} = 4$

(b) $\lim\limits_{x \to (-1)} \dfrac{x^2}{x+1}$. DNE because you are getting a number divided by zero.

(c) $\lim\limits_{x \to \infty} \dfrac{3x^4 - 3x}{7x^2 - 11x^4 + 4} =$ -3/11

10. Let $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x \le 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

Find the value of the following limits. If the limit does not exist, write DNE and explain why.

(a) $\lim\limits_{x \to (-1)^-} f(x) = 1$

(b) $\lim\limits_{x \to (-1)^+} f(x) = 1$

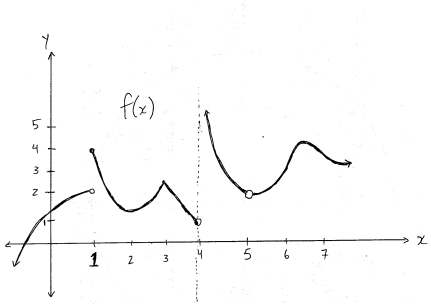
(c) $\lim\limits_{x \to 2^-} f(x) = 4$

(d) $\lim\limits_{x \to 2^+} f(x) = 3$

(e) $\lim\limits_{x \to 1^-} f(x) = 1$

(f) $\lim\limits_{x \to \infty} f(x) = 3$

11. Find the given limits for the following diagram. If the limit does not exist, write DNE and explain why.



(a) $\lim_{x\to 1^+} f(x) = 4$

(b) $\lim_{x\to 4^-} f(x) = 1$

(c) $\lim_{x\to 4^+} f(x) =$ DNE because the graph is going to infinity.

(d) $\lim_{x\to 5} f(x) = 2$

12. Use the limit definition of the derivative to calculate $f'(x)$ if $f(x) = \dfrac{1}{2x+3}$.

Ans: You need to do $f(x+h) =$, etc. You answer should be $f'(x) = \dfrac{-2}{(2x+3)^2}$.

13. Find the derivative of each function: NOTE: Some of these answers are in simplified form

(a) $f(x) = (3x^4 - 7)(x^2 + 9)$. Use the product rule to get $18x^5 + 108x - 14x$

(b) $f(x) = (x^3 - 7x + 9)^7$. Chain rule: Ans: $7(x^3 - 7x + 9)^6(3x^2 - 7)$

(c) $f(x) = \left(\dfrac{x^3 - 9}{x + 4}\right)^3$. Ans: $\dfrac{3(x^3 - 9)^2(2x^3 + 12x^2 + 9)}{(x+4)^4}$

(d) $f(x) = (x + 7)^4(3x^2 - 4)^2$. Ans: $4(x + 7)^3(3x^2 - 4)(6x^2 + 21x - 4)$

14. The quantity x of TV sets demanded each week is related to the wholesale price by the equation $p = -0.006x + 180$. The weekly total cost for producing x sets is given by $C(x) = 0.00002x^3 - 0.02x^2 + 120x + 60,000$.

(a) Find the revenue function $R(x)$ and the profit function $P(x)$.

Ans: $R(x) = -0.006x^2 + 180x$, $P(x) = -0.00002x^3 + 0.014x^2 + 60x - 60,000$.

(b) Compute the marginal revenue, cost, and profit functions.

Ans: $R'(x) = -0.012x + 180$, $C'(x) = 0.00006x^2 - 0.04x + 120$, $P'(x) = -0.00006x^2 + 0.028x + 60$

(c) Compute $R'(2000)$, $C'(2000)$, and $P'(2000)$ and interpret your results. What does that information tell the company about how many TV sets they should produce?

Ans: $R'(2000) = 156$, $C'(2000) = 280$, and $P'(2000) = -124$. At production of 2000 TV sets, the costs still exceed the revenues and the next TV made will not give any profit. Producing only 2000 TV sets is not good for profits.

15. The number of people receiving disability benefits from 1990 through 2000 is approximated by the function

$$N(t) = 0.00037t^3 - 0.0242t^2 + 0.52t + 5.3 \qquad (0 \leq t \leq 10)$$

where $N(t)$ is measured in units of a million and t is measured in years with $t = 0$ being 1990. Compute $N(8)$, $N'(8)$, and $N''(8)$ and interpret your results. What does that information tell you about what was happening with disability benefits at that time, and what might it imply for the future?

Ans: 8.1 million, 203,840, and -30,640. This means that in 1998, 8.1 million people were receiving disability benefits, and that number was increasing by 203,840 a year. Since the second derivative is negative, though, the amount of the increase is going to go down in the next year, by about 30,640 people. So even though the number of people receiving disability benefits will be increasing, the amount of the increase was going down in 1998.